Maths for Physics course

Although 'A' level Maths is not a requirement for 'A' level Physics it is critical for your success at 'A' level Physics that you are very confident at the Mathematical techniques in this booklet.

In the first few weeks of year 12 at King Edward VI you will be given a 'Maths for Physics' test. This will include **rearranging equations**, trigonometry, Pythagoras' theorem, indices, standard form, significant figures, substituting numbers into formula, prefixes (eg. What do Mega and nano mean) and graphs. This booklet has been prepared in order to help you with the test.

Your Task

1. Read through **ALL** of the booklet
2. Read through the booklet again, completing the following student tasks as you get to them.
   - Student task 1.1
   - Student task 1.2
   - Student task 1.3
   - Student task 1.4
   - Student task 2.4
   - Student task 2.7
   - Student task 4.3
   - Student task 4.4 & 4.5
3. If there is anything that you don't understand you need to research the problem.
4. Revise, using the booklet, for the test in the first few weeks back.
5. Hand in your completed exercises to your Physics teacher in your **FIRST** lesson back.
6. Past experience has shown us that student's that don't do well on the 'Maths for Physics' test find the Physics 'A' level very difficult. Please work hard at mastering the techniques in this booklet!

1.3 THE BASIC RULES OF ALGEBRA AND ARITHMETIC

In order to be able to manipulate algebraic equations as well as carrying out numerical calculations, you must know and understand the basic rules of algebra. Four useful examples are shown in Table 1.2.

Table 1.2 Simple algebraic rules

	Example
$by = y + y + ...$(b times)	$3y = y + y + y$
$ay + by = (a + b)y$	$2y + 3y = 5y$
$\frac{1}{\left(\frac{a}{y}\right)} = \frac{y}{a}$	$\frac{1}{\left(\frac{2}{y}\right)} = \frac{y}{2}$
$\frac{\left(\frac{b}{z}\right)}{\left(\frac{a}{y}\right)} = \frac{by}{az}$	$\frac{\left(\frac{3}{z}\right)}{\left(\frac{2}{y}\right)} = \frac{3y}{2z}$

1.4 HANDLING INDICES

y^a means y multiplied by itself a times. For example, 4^3 means $4 \times 4 \times 4 = 64$.

a is called the index, or the power to which y is raised. (We say, for example 'y to the power of a' or '4 to the power of 3'.)

Many scientific equations contain indices. The rules for manipulating indices are shown in Table 1.3.

Table 1.3 Handling Indices

	Algebraic example	Numerical example
$y^{-n} = \frac{1}{y^n}$	$y^{-3} = \frac{1}{y^3}$	$4^{-3} = \frac{1}{4^3} = \frac{1}{64} = 0.0156$
$y^{\frac{1}{a}} = \sqrt[a]{y}$	$y^{\frac{1}{2}} = \sqrt{y}$	$4^{\frac{1}{2}} = \sqrt{4} = 2$
$y^a + y^b$ cannot be simplified		$4^2 + 4^3 = 16 + 64 = 80$
$y^a \times y^b = y^{a+b}$	$y^2 \times y^3 = y^5$	$4^2 \times 4^3 = 4^5 = 1024$
$\frac{y^a}{y^b} = y^{a-b}$	$\frac{y^2}{y^3} = y^{2-3} = y^{-1}$	$\frac{4^2}{4^3} = 4^{-1} = \frac{1}{4} = 0.25$
$(y^a)^b = y^{a \times b}$	$(y^2)^3 = y^6$	$(4^2)^3 = 4^6 = 4096$
$y^{\frac{b}{a}} = \sqrt[a]{y^b}$	$y^{\frac{3}{2}} = \sqrt[2]{y^3}$	$4^{\frac{3}{2}} = \sqrt[2]{4^3} = 8$
$y^0 = 1$		$4^0 = 1$

✍ ***Student task 1.1***

a) Simplify the following:

i) $y^6 \times y^7$ ii) $y^8 \div y^5$

iii) $(y^3)^4$ iv) $3y^2 \times 4y^3$

b) Calculate the values of the following:

i) 3^4 ii) $8^{\frac{1}{3}}$ iii) 5^{-2}

iv) $9^{\frac{3}{2}}$ v) $16^{-\frac{1}{2}}$ vi) 6^0

1.5 CHANGING THE SUBJECT OF AN EQUATION

In the equation $v = u + at$, v is known as the subject of the equation. You can calculate a value for v if you know the values of u, a and t by substituting these values into the equation. However, you may need to calculate the value of t, for example, so you would need to make t the subject of the equation.

Changing the subject of an equation is based on the general principle:

> **Whatever change you make to one side of the equation, you must make the same change to the other side.**

The following examples should make this clear.

i) Addition and subtraction

The equation for resistors in series is:

$$R_T = R_1 + R_2$$

To make R_1 the subject of the equation, we subtract R_2 from the right hand-side, leaving only R_1, so we also subtract R_2 from the left-hand side, giving:

$$R_T - R_2 = R_1$$

or

$$R_1 = R_T - R_2$$

Effectively, we have moved R_2 to the other side of the equation and changed its sign from positive to negative.

> **If something which is added to one side of an equation is moved to the other side, it changes its sign from positive to negative. Conversely, if something which is subtracted from one side of an equation is moved to the other side, it changes its sign from negative to positive.**

> ✍ ***Student task 1.2***
>
> a) Make u the subject of $v = u + at$
>
> b) Make E_k the subject of $hf = \varphi + E_k$
>
> c) Make $\frac{1}{v}$ the subject of $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

ii) Multiplication and division

This is by far the most common kind of equation in Physics.

The equation for power in an electric circuit is:

$$P = VI$$

To make V the subject of the equation, we divide the right hand side by I, so we must also divide the left hand side by I giving:

$$\frac{P}{I} = V$$

Effectively we have moved I to the other side of the equation and changed its sign from 'multiply' to 'divide'.

> **If something which multiplies one side of an equation is moved to the other side, it divides that side of the equation (it moves from being a *numerator* to being a *denominator*). Conversely, if something which divides one side of an equation moves to the other side, it multiplies that side of the equation.**

> ✍ ***Student task 1.3***
>
> a) Make B the subject of $F = BIl$
>
> b) Make ρ the subject of $R = \frac{\rho l}{A}$
>
> c) Make V the subject of $C = \frac{Q}{V}$

iii) Squares and square roots

If $$x^2 = a + b$$

then to find x we must find the square root of both sides, so

$$x = \sqrt{a + b}$$

If $$\sqrt{y} = c + d$$

then to find y we must square both sides, so

$$y = (c + d)^2$$

iv) Further examples

Example a)

The efficiency of a heat engine is given by the equation:

$$E = \frac{T_1 - T_2}{T_1}$$

Make T_2 the subject of the equation.

Since the whole of the right-hand side of the equation is divided by T_1, the first step is to take this T_1 to the left-hand side:

$$ET_1 = T_1 - T_2$$

Now $-T_2$ can be taken to the left, and ET_1 to the right:

$$T_2 = T_1 - ET_1$$

Finally, T_1 could be taken out as a common factor on the right:

$$T_2 = T_1(1 - E)$$

If we wanted to make T_1 the subject of the equation, we would have to go through the same steps, and finally take the $(1 - E)$ to the left-hand side, leaving just T_1 on the right:

$$\frac{T_2}{(1 - E)} = T_1$$

> **✻ *Pitfall 1.1***
>
> In this example, it is easy to forget that T_2 is divided by T_1, resulting in the following:
>
> $$E = \frac{T_1 - T_2}{T_1} \quad \Rightarrow \quad E + T_2 = \frac{T_1}{T_1} = 1$$
>
> which is clearly nonsense!

Example b)

The frequency of oscillation of a mass on a spring is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Make k the subject of the equation.

Since m is inside the square root sign, we start by moving 2π to the left-hand side:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad 2\pi f = \sqrt{\frac{k}{m}}$$

Now both sides of the equation can be squared to remove the square root:

$$2\pi f = \sqrt{\frac{k}{m}} \quad \Rightarrow \quad (2\pi f)^2 = \frac{k}{m}$$

Finally, m can be moved to the opposite side of the equation.

$$(2\pi f)^2 = \frac{k}{m} \quad \Rightarrow \quad m(2\pi f)^2 = k$$

> **✻ *Pitfall 1.2***
>
> Beware of accidentally removing a variable from within a square root. A possible mistake in the previous example is:
>
> $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad mf = \frac{1}{2\pi}\sqrt{k}$$
>
> However, the following is correct, although not particularly easy to work with:
>
> $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \Rightarrow \quad \sqrt{m}\,f = \frac{1}{2\pi}\sqrt{k}$$
>
> However, it is possible to confuse the following:
>
> $$\sqrt{m}\,f \quad \text{with} \quad \sqrt{mf}$$
>
> especially with hasty handwriting. It is much safer to write this as:
>
> $$f\sqrt{m}$$

Example c)

Make u the subject of the following equation of motion:

$$v^2 = u^2 + 2as$$

The necessary stages are:

$$v^2 = u^2 + 2as \quad \Rightarrow \quad u^2 = v^2 - 2as$$

$$u^2 = v^2 - 2as \quad \Rightarrow \quad u = \sqrt{v^2 - 2as}$$

> **✻ *Pitfall 1.3***
>
> $\sqrt{v^2 - 2as}$ does not equal $v - \sqrt{2as}$

Example d)

It is often necessary to move several variables to achieve what is required, but the principles remain the same.

The magnetic field in a long solenoid is given by:

$$B = \frac{\mu_0 NI}{l}$$

To make I the subject of the equation, we move l to the left, where it becomes part of the numerator ('from the bottom to the top of the fraction'), while $\mu_0 N$ moves to the left to become part of the denominator ('from the top to the bottom of the fraction'), giving:

$$\frac{Bl}{\mu_0 N} = I \quad \text{or} \quad I = \frac{Bl}{\mu_0 N}$$

✍ Student task 1.4

a) Make l the subject of $R = \frac{\rho l}{A}$

b) Make x the subject of $\frac{\lambda}{x} = \frac{s}{l}$

c) Make v the subject of $F = \frac{mv^2}{r}$

d) Make r the subject of $F = \frac{Gm_1 m_2}{r^2}$

e) Make E the subject of $c = \sqrt{\frac{E}{\rho}}$

f) Make T the subject of $f = 2\pi\sqrt{\frac{T}{\mu}}$

g) Make L the subject of $f_r = \frac{1}{2\pi\sqrt{LC}}$

1.6 SIMULTANEOUS EQUATIONS

If you have two equations relating two different unknown quantities, these equations are known as 'simultaneous'. They have a limited use in A-level Physics.

A simple example of a pair of simultaneous equations is:

$$x + 7y = 38$$
$$x + 3y = 18$$

To solve these equations (that is, find the values of x and y), we need to eliminate one of the variables; in this case it is easiest to eliminate x by subtracting the second equation from the first:

$$\begin{array}{c} x + 7y = 38 \\ x + 3y = 18 \\ \hline (x - x) + (7y - 3y) = (38 - 18) \\ 4y = 20 \\ \therefore y = 5 \end{array}$$

This value for y can now be substituted back into the first of the pair of equations to find a value for x:

$$x + (7 \times 5) = 38$$
$$x + 35 = 38$$
$$\therefore x = 3$$

Example

Two cars are moving along a road; car 1 is moving at a steady speed of 20 m/s; car 2 is 150 m in front of car 1 and moving at a steady speed of 15 m/s.

How much time passes before car 1 catches up with car 2, and how far will it have travelled?

Let s be the displacement when they meet, measured from the starting position of the first car, v_1 and v_2 the respective speeds of the cars and t the time at which they meet.

For car 1: $s = v_1 t$ $\quad \therefore s = 20t$

For car 2: $s = v_2 t + 150$ $\quad \therefore s = 15t + 150$

Subtracting the two equations gives:

$$0 = 5t - 150$$
$$5t = 150$$
$$\therefore t = 30$$

Car 1 passes car 2 after 30 seconds.

By substituting this value into the first equation, we can see that car 1 travels

$$20 \text{ m/s} \times 30 \text{ s} = 600 \text{ m}$$

before overtaking car 2.

1.7 QUADRATIC EQUATIONS

An equation which contains both x^2 and x is known as a quadratic equation. Quadratic equations do not occur very often in A-level Physics.

The general form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

where x is a variable and a, b and c are constants.

The general solution to this equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice that there will usually be two possible values for x.

A simple example of a quadratic equation is:

$$2x^2 - 11x + 4 = 0$$

Here a = 2, b = -11 and c = 4.

The two possible values of x are given by:

$$x = \frac{11 \pm \sqrt{121 - 32}}{4}$$

$$\therefore x = \frac{11 + \sqrt{89}}{4} \quad \text{or} \quad \frac{11 - \sqrt{89}}{4}$$

Hence $x = 5.11$ or $x = 0.39$

(You may be aware of another method of solving quadratic equations involving factorisation. The figures involved in the real world of Physics rarely make this method feasible).

The most common quadratic equation to occur in A-level Physics is the equation of motion $s = ut + ½at^2$.

Example

A ball is launched vertically upwards with a speed of 150 m/s. How much time elapses before it reaches a height of 500 m?

Table of information:

initial speed $u = 150$ m/s

acceleration $a = -10$ m/s²

displacement $s = 500$ m

(Note: if the initial velocity is positive, the acceleration is negative since it is in the opposite direction.)

Calculation:

Using $s = ut + ½at^2$

$$500 = 150t - (½ \times 10t^2)$$

$$500 = 150t - 5t^2$$

Rearranging the equation:

$$5t^2 - 150t + 500 = 0$$

and dividing all through by 5:

$$t^2 - 30t + 100 = 0$$

Using the general equation to find the values of t:

$$t = \frac{-(-30) \pm \sqrt{900 - 400}}{2}$$

$$t = \frac{30 + \sqrt{500}}{2} \quad \text{or} \quad t = \frac{30 - \sqrt{500}}{2}$$

$$\therefore t = 26.2 \text{ or } 3.8$$

The time taken is either 3.8 s or 26.2 s. The first value is the time taken to pass the 500 m height on the way up, and the second the time to pass on the way down. A little thought about the situation is needed to interpret the Mathematics.

✻ *Pitfall 1.4*

Beware of meaningless negative answers to quadratic equations. For example:

A ball is thrown down with an initial velocity of 15 m/s from a tower 60 m tall. How much time elapses before it hits the ground?

A similar calculation to the previous example using these figures gives times of 2.3 s or -5.3 s. The second answer is nonsense in this context.

✍ *Student task 1.5*

a) Solve the problem in Pitfall 1.4 to satisfy yourself that the times quoted are correct.

b) Can you think of what the -5.3 s obtained as the second answer in this example could represent?

1.8 DEDUCTIONS FROM EQUATIONS

This section is concerned with the effect on one variable of changing another related variable, as in the following examples.

Example 1

$$F = ma$$

If the force F doubles and the mass m remains constant, then the acceleration a must double as well for the equation to remain balanced. If the value of the left-hand side of an equation doubles, then the value of the right-hand side must also double.

Similarly, if F trebles, a trebles; if F halves, a halves, and so on. We say that F and a are directly proportional to each other.

Example 2

$$stress = \frac{force}{area}$$

If the *area* doubles and the force remains constant, the value of the right-hand side halves, since the *force* is divided by a number twice as large as before. Therefore the value of *stress* halves.

Here we say that the *stress* and *area* are inversely proportional to each other.

Example 3
The equation relating the period T of a pendulum to its length l is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where g is the gravitational field strength.

If the length l doubles, the right-hand side of the equation becomes

$$2\pi\sqrt{\frac{2l}{g}}$$

that is, it is $\sqrt{2}$ ($\approx$ 1.4) times larger than before. Therefore the period T increases by a factor of $\sqrt{2}$.

Here we say the period is proportional to the square root of the length; if the length doubles, the period increases by a factor $\sqrt{2}$; if the length quadruples the period doubles, and so on.

Example 4
The equation for the force between two bodies with electric charges Q_1 and Q_2 is given by:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

If the distance r doubles, the right-hand side of the equation becomes

$$\frac{Q_1 Q_2}{4\pi\varepsilon_0 (2r)^2}$$

that is, it is $(\frac{1}{2})^2 = \frac{1}{4}$ of its original value; therefore the force F reduces to ¼ of its original value.

In this case the force F is inversely proportional to the square of the distance r.

Example 5

In some situations, more than one variable is changed. For example, suppose a piece of wire has its length doubled and its cross-sectional area halved; what will be the effect on its resistance?

The relevant equation is

$$R = \frac{\rho l}{A}$$

If l is doubled and A is halved, the right-hand side becomes

$$\frac{\rho(2l)}{(\frac{1}{2}A)} \quad \text{or} \quad \frac{4\rho l}{A}$$

which is 4 times larger than before. Therefore the resistance increases by a factor of 4.

✍ *Student task 1.6*

A piece of wire has both its length and its diameter doubled; what will be the effect on its resistance?

(First show that if the diameter doubles the cross-sectional area quadruples, then use the same equation as in the example above the calculate the effect on the resistance.)

2.3 STANDARD FORM

Many of the numbers we meet in Physics are either very large or very small.

For example, the mass of the earth is:

5 980 000 000 000 000 000 000 000 kg

while the charge on an electron is:

0.000 000 000 000 000 000 16 C

It is clearly ludicrous to try to do calculations with numbers written like this. You cannot key them into your calculator for a start, and it would be very easy to make a mistake with the number of zeros. In order to overcome this problem, we write numbers in standard form. A number is expressed as a number between 1 and 10 multiplied by an appropriate power of 10.

For example, 347 can be written as 3.47 × 100, or 3.47×10^2.

3.47×10^2 is known as standard form.

Using standard form, the mass of the earth can be written as 5.98×10^{24} kg, and the charge on an electron as 1.6×10^{-19} C.

This is clearly a much more convenient way to write such numbers, but there are other good reasons for using standard form, outlined in the following sections. It is very important that you should be able to handle standard form easily.

2.4 CONVERTING TO STANDARD FORM

To convert 37 800 to standard form.

The number between 1 and 10 which is needed is 3.78.

37 800 is 3.78 × 10 000. 10 000 is 10^4 (see Table 2.1) so 37 800 in standard form is 3.78×10^4.

An alternative approach is to say that to go from 3.78 to 37 800, the numbers must move 4 places to the left, as follows:

3.78
37.8
378.0
3 780.0
37 800.0

Therefore the power of 10 required is 4, and 37 800 in standard form is 3.78×10^4.

To convert 0.0052 to standard form.

The number between 1 and 10 which is needed is 5.2. 0.0052 is 5.2 × 0.001. 0.001 is 10^{-3}, so 0.0052 in standard form is 5.2×10^{-3}.

Alternatively, to go from 5.2 to 0.0052 the numbers must be moved 3 places to the right, therefore the required power is −3, and 0.0052 in standard form is 5.2×10^{-3}.

Table 2.1

10^6 =	1 000 000
10^4 =	10 000
10^3 =	1000
10^1 =	10
10^0 =	1
10^{-1} =	0.1
10^{-3} =	0.001
10^{-6} =	0.000 001

Student task 2.4

Convert the following numbers to standard form:

a) 3470
b) 68 000 000
c) 27
d) 0.594
e) 0.000 92
f) 264.2

2.6 SIGNIFICANT FIGURES

The significant figures in a number are all the digits except any zeros before the first non-zero digit.

For example:	42, 4.2, 0.0042 and 4.2×10^6	all have 2 significant figures
	402 and 4.02	have 3 significant figures

The number of significant figures has an important meaning to a scientist. It gives you some idea of the precision or reliability of that number.

For example, you might measure the length of a piece of wood quite roughly as being 5 cm; this means you are certain of its length to the nearest centimetre - its actual length could be anywhere between 4½ and 5½ cm.

If you measure the length more carefully, you might be able to give the measurement to 2 significant figures, 5.1 cm, for example. This means you are now more certain of the length than you were before; you are probably certain to the nearest 0.1 cm. (Section 2.9, on uncertainties, goes into more detail).

3 significant figures, such as 5.13 cm, indicates an even greater precision.

Notice that the numbers 8, 8.0 and 8.00 do not all mean the same; 8.00 (3 significant figures) implies a much greater precision than 8 (1 significant figure).

A problem can arise over interpreting some numbers if we do not use standard form. Suppose we measure the length of a road as 3200 m. It is not clear whether you have measured to the nearest 100 m, and are giving a measurement to 2 significant figures, or to the nearest metre and all 4 figures are significant. If you write the measurement using standard form, 3.2×10^3 m (2 significant figures) or 3.200×10^3 m (4 significant figures), the precision of the measurement is immediately clear.

2.7 SIGNIFICANT FIGURES AND CALCULATIONS

The answer to a calculation cannot be any more precise than the least reliable piece of data used for that calculation. It is therefore important that your answer to any calculation is rounded to the correct number of significant figures; as a good rule of thumb, this should be the same number of significant figures as your least reliable piece of data. For the data which occurs in typical A-level questions, 2 significant figures is usually right; you may lose marks in an examination if you do not round your answer to a sensible number of significant figures.

Example

A copper block has a mass of 246.3 g. It measures 2.2 cm by 3.0 cm by 4.5 cm. What is the density of the copper?

$$density = \frac{mass}{volume}$$

$$density = \frac{246.3 \text{ g}}{2.2 \text{ cm} \times 3.0 \text{ cm} \times 4.5 \text{ cm}}$$

$$density = 8.292\,929\,3 \text{ g cm}^{-3}$$

according to a calculator.

But this implies a precision far greater than any of the measurements that were made. The lengths were measured to 2 significant figures, so the answer must be rounded to 2 significant figures, giving a density of 8.3 g cm^{-3}.

Notice that the mass of the block was measured with a greater precision than was necessary. It is usually a waste of time making one measurement much more precise than any of the others.

✍ ***Student task 2.6***

Try repeating the calculation with a mass of 246 g, or even 245 g.

2.8 UNITS AND STANDARD FORM

Quantities are often quoted in standard multiples or divisions of the basic units. The standard prefixes, together with the factor by which they multiply the basic unit, are listed in table 2.2 below.

Table 2.2

Name	Abbreviation	Multiplying factor
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

For example

1 millimetre = 1 thousandth of a metre

$= 1 \times 10^{-3}$ m

4.6 kW $= 4.6 \times 10^{3}$ W

✍ *Student task 2.7*

Write the following in standard form.

a) 7.21 μV

b) 42 GW

c) 0.39 nF

d) 592 MJ

e) 0.019 pA

Quantities often have to be written in terms of the basic unit before a calculation can be performed. This can easily be done by inserting the appropriate power of 10 in place of the prefix, and is best done by writing out a table of the relevant information before you start, as in the example in the next column. (It is a good habit to write out a table of information for anything but the simplest calculation whether or not you have to do any unit conversions.)

Example

An electric power cable has a diameter of 6.0 mm and is made of a material of resistivity 27 nΩm. What is the resistance of a 1.0 km length of the cable?

Table of information:

Diameter = 6.0 mm	$= 6.0 \times 10^{-3}$ m
Hence radius	$= 3.0 \times 10^{-3}$ m
Length = 1.0 km	$= 1.0 \times 10^{3}$ m
Resistivity = 27 nΩm	$= 27 \times 10^{-9}$ Ωm
Resistivity	$= 2.7 \times 10^{-8}$ Ωm

Calculation:

$$\text{Cross section area (CSA)} = \pi r^2$$

$$CSA = \pi \times 3.0 \times 10^{-3} \times 3.0 \times 10^{-3}\ \text{m}^2$$

$$CSA = 2.827 \times 10^{-5}\ \text{m}^2$$

$$\text{Using } R = \frac{\rho l}{A}$$

$$\left(\text{Resistance} = \frac{\text{resistivity} \times \text{length}}{\text{area}}\right)$$

$$R = \frac{2.7 \times 10^{-8}\ \Omega\text{m} \times 1.0 \times 10^{3}\ \text{m}}{2.827 \times 10^{-5}\ \text{m}^2}$$

$$R = 0.96\ \Omega$$

The resistance is 0.96 Ω.

✻ *Pitfall 2.7*

Mistakes are often made in converting areas and volumes into the basic units, such as square millimetres into square metres.

Note

10^4 square centimetres = 1 square metre.

10^6 square millimetres = 1 square metre

10^6 cubic centimetres = 1 cubic metre

10^9 cubic millimetres = 1 cubic metre

✍ *Student task 2.8*

Calculate the resistance between the faces of a wafer of pure silicon of thickness 5 mm and cross section area 8.0 cm². The resistivity of silicon is 60 Ωm.

(Unlike the question above, you have been told the area, not the diameter.)

A practical activity: The pendulum

Testing a relationship

A graph can be used to test the relationship between two variables and confirm the value of a constant. In this case it is useful if the relationship between the variables can be arranged as the equation of a straight line. We can then see how closely the plotted points lie on a straight line and this tells us if we have a linear relationship.

WORKED EXAMPLE

This practical activity is to show whether the equation relating the period (the time for one swing) of a pendulum to its length is correct. We would need to measure the period, T, for different lengths of pendulum, l, and show that our data are consistent with the equation for the period:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where g is the acceleration due to gravity.

If we plot T against l, the graph will be a curve, and we will not be able to tell if the equation is correct. If we change the equation to give a linear relationship then we will be able to tell from the graph if the equation is correct, because if we get a straight line the relationship is correct. Squaring both sides of the equation gives:

$$T^2 = 4\pi^2\frac{l}{g}$$

Compare this with the straight line equation: $y = mx + c$

This equation will give a straight line if we plot T^2 against l, that is: $T^2 = y$, $l = x$

and then $c = 0$ and $m = \frac{4\pi^2}{g}$.

So we should get a straight line and, as $c = 0$, this means it should go through the origin. When a straight-line graph goes through the origin this means that the variables are proportional to each other. This can be written $T^2 \propto l$ (if there is a non-zero value for the constant c the graph doesn't pass through the origin and the relationship is linear, but not proportional).

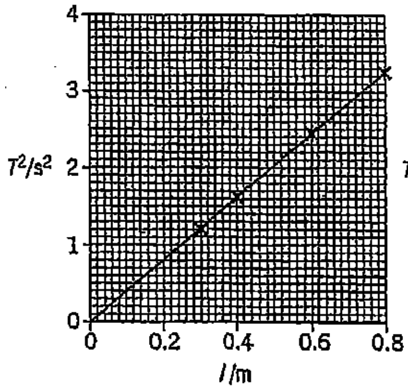
m is the gradient of the straight line and from this we can calculate a value for g by rearranging the equation for m to give $g = 4\pi^2/m$. We can compare our value of g with the accepted value of $9.81\,\mathrm{m\,s^{-2}}$.

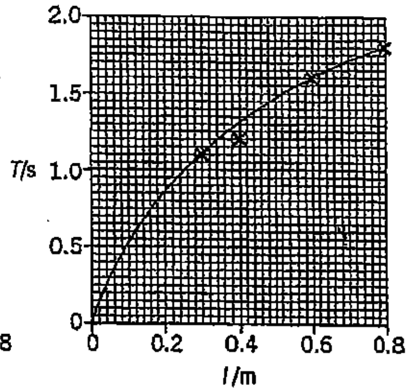
To reduce the uncertainty in the data we can time the pendulum for 20 swings. This will be a larger time and so the percentage uncertainty in the measurement will be less.

Graphs of T and T^2 against l for a pendulum

Time for 20 swings/s	T/s	T^2/s^2	l/m
22	1.1	1.21	0.3
26	1.2	1.69	0.4
32	1.6	2.56	0.6
36	1.8	3.24	0.8

l = pendulum length
T = period





4.4 THE GRADIENT OF A GRAPH

The gradient or slope of a graph (that is, how 'steep' the graph is,) measure the rate at which the 'y' variable is changing with respect to the 'x' variable.

The gradient is simply a change in the y value (denoted by Δy) divided by the corresponding change in the x value (denoted by Δx) as shown in Figure 4.12.

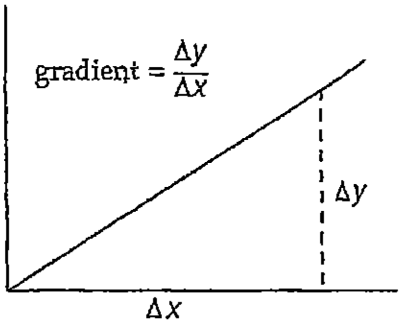


Figure 4.12

In Figure 4.13, the gradient is $6 \div 2 = 3$. Notice that in this graph the value of y is always 3 times the value of x; in other words, the equation of the line is $y = 3x$.

In general, the equation of a straight line through the origin is $y = mx$, where m is the gradient of the line. (See Section 4.1.)

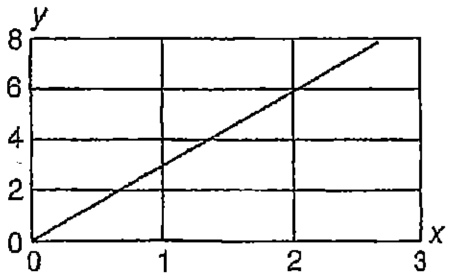


Figure 4.13

A graph such as that in Figure 4.14, which slopes in the opposite direction to that in the previous diagram, has a negative gradient. The gradient of this graph is -0.4

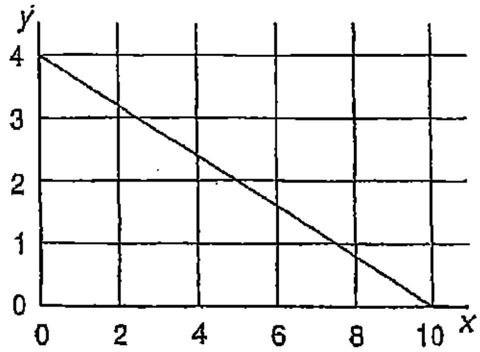
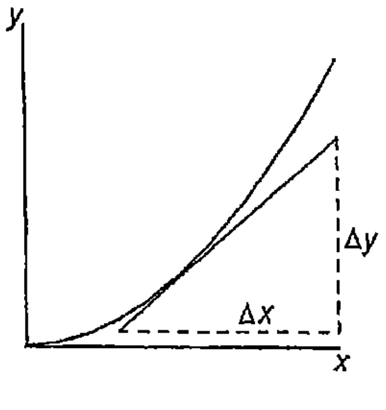


Figure 4.14

> ✍ *Student task 4.2*
>
> What is the equation of the line shown in Figure 4.14?
>
> (It will be of the form $y = mx + c$, since the line does not go through the origin. c is the intercept on the y-axis. See Pitfall 4.1.)

The gradient of a curved graph clearly changes - it is different at all points on the graph. In this case it is necessary to draw a tangent to the curve at the place in which you are interested, and find the gradient of that tangent, as shown in Figure 4.15.

Figure 4.15



The gradient of a graph often has a physical significance and can provide useful information about a situation. For example, the gradient of a graph of velocity against time is equal to the acceleration; the gradient of a graph of electric potential against distance is equal to the electric field.

✍ *Student task 4.3*

a) Calculate the gradients of the graphs shown in Figures 4.15 and 4.16.

b) Estimate the gradient of the graph in Figure 4.17 at point P and at point Q.

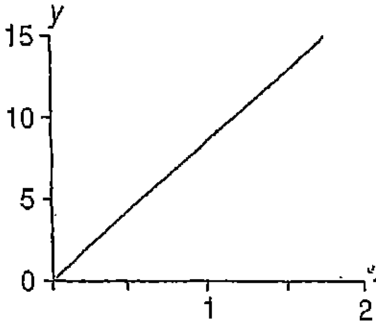


Figure 4.15

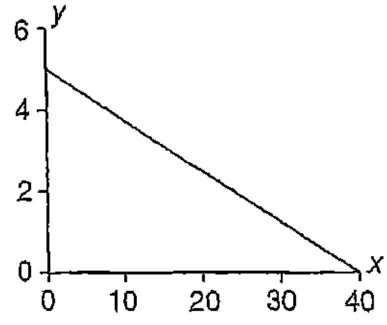


Figure 4.16

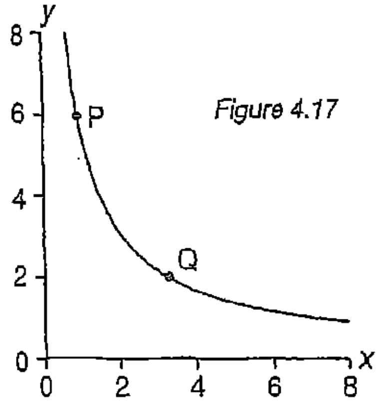


Figure 4.17

Units, tabulation, graphs and significant figures

Labelling in table headings and on graph axes

The convention we prefer is

variable symbol – solidus – unit

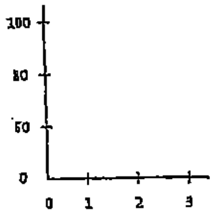
Candidates should be discouraged from naming the variable (which in any case will be defined in the question), in full.

eg 'V/mV' is simple and effective, 'output pd of solar cell in millivolts' is not

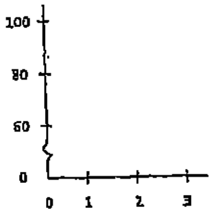
The solidus indicates the division of a physical quantity by its unit, thus what follows is a pure number.

eg 'V/mV = 340' literally means 340 is the value of V divided by mV; using 'V/mV' as a table heading is logical and correct in a way that V(mV) is not.

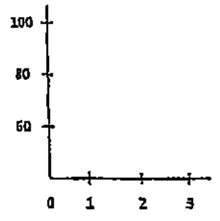
Marking the origin correctly on a graph, eg PHAB3X Sec A Part 1 Q2(b)



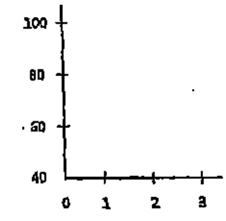
Unacceptable:
the marking of the origin as above produces a non-linear scale which will always be penalised.



Solution:
use of the broken scale convention resolves the problem but watch out if a gradient calculation is then required.



Unacceptable:
leaving an origin unmarked on either axis will not be accepted; the scale will still be treated as non-linear since the origin is now ambiguous.



Solution:
use of a false origin is acceptable but candidates should be careful if they are then asked to calculate the gradient.

Please turn over for task 4.4 & 4.5

Student task 4.4 – using special relativity equations

1) $t = t_o (1 - v^2/c^2)^{-1/2}$

Re-arrange for a) t_o

b) v

c) Calculate t if, $t_o = 2.20 \times 10^{-6}$s

$c = 3 \times 10^8$m/s

$v = 2.97 \times 10^8$m/s

2) $L = L_o (1 - v^2/c^2)^{1/2}$

Re-arrange for a) L_o

b) v

c) Calculate L if, $L_o = 60$m

$c = 3 \times 10^8$m/s

$v = 2.94 \times 10^8$m/s

3) $m = m_o (1 - v^2/c^2)^{-1/2}$

Re-arrange for a) m_o

b) v

c) Calculate m if, $m_o = 1.67 \times 10^{-27}$Kg

$c = 3 \times 10^8$m/s

$v = 2.7 \times 10^8$m/s

Student task 4.5 - Further practise at re-arranging equations

1) $d\sin\theta = n\lambda$ re-arrange for a) d b) $\sin\theta$ c) λ
2) $V = Q/4\pi\varepsilon_0 r$ re-arrange for a) ε_0 b) Q c) r
3) $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ re-arrange for a) r b) Q_1
4) $\sin\theta_1/\sin\theta_2 = n_2/n_1$ re-arrange for a) $\sin\theta_1$ b) $\sin\theta_2$ c) n_2 d) n_1
5) $v^2 = u^2 + 2as$ re-arrange for a) a b) s c) u
6) $T = 1/f$ re-arrange for f
7) $N_S/N_P = V_S/V_P$ re-arrange for a) N_S b) N_P c) V_S d) V_P
8) $F = GMm/r^2$ re-arrange for a) M b) r^2 c) r
9) $\rho = RA/l$ re-arrange for a) R b) A c) l
10) $P = I^2R$ re-arrange for a) I^2 b) I c) R